Long Distance Processes in Tone and Subsequentiality

Tajudeen Mamadou

Rutgers

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Roadmap

- What is the right characterization for tonal processes?
  - The key notion here is determinism
  - Introduce a new class of functions (IML functions)
  - Model them with a restrictive type of finite state transducers
  - Show the new class's empirical coverage
- Carve out a deterministic class for tone processes, excluding known pathologies

[Tonal functions are maximally Input or Output Melody Local]

Background

- Most phonological processes are **Subsequential:**
  - A property of **string functions**

[Payne, 2017, Chandlee, 2014, Heinz and Lai, 2013, Mohri, 1997]

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$\underbrace{\boxed{a} \longrightarrow \text{TARGETS}}_{\textbf{Left-Subseq}}$ *or* $\underbrace{\text{TARGETS} \longleftarrow \boxed{a}}_{\textbf{Right-Subseq}}$ ✓

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$$\underbrace{\boxed{a} \longrightarrow \text{TARGETS} \longleftarrow \boxed{b}}_{\textbf{Non-Subseq}} \qquad ✗$$

[Payne, 2017, Chandlee, 2014, Heinz and Lai, 2013, Mohri, 1997]

Tone & Non-subsequentiality

- Tone includes Unbounded Circumambient processes (UC)

[Hyman, 2011, Jardine, 2016]

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a.	/mutéma+bisikí/	[mutémá+bísíkí]	‘log chopper’
b.	Unattested	*[mutéma+bisikí]	_

LHLLLLL	→	L**H**LLLLL
LHLL(...)LHL	→	L**HHH(...)HH**L

[Hyman and Katamba, 2010]

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- UC processes are non-subsequential
- We can make them deterministic with a two tier representation

[Hyman and Katamba, 2010, Jardine, 2016]

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[Rawski and Dolatian, 2020, Dolatian and Rawski, 2020]

Input Melody Local (IML)

- IML functions extend subsequentiality to UTP-like processes
- Represent a sub-class of a recently introduced Multi-Input Strictly Local (MISL) class
- Intuitively, we are enriching the representation while maintaining a notion of autosegmental locality

[Odden, 1994]

Input Melody Local (IML)

- **Two key components**:
  - **A Melody Function**: Only retains one symbol in each span of tones (like the OCP); assumes underlying associations

    `mel`(LHLLLLH)=LHLH

[Jardine, 2020]

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Input

melody	L	H	L					H
timing	L	H	L	L	L	L	L	H

IML Functions

- Intuitively, we want our function to be computed deterministically as follows:

Input

melody	L	H	L					H
timing	L	H	L	L	L	L	L	H

Output : L

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Output : LHH

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- In general, multi-tier function takes an input of the type $\langle \texttt{mel}(w), w \rangle$:

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- E.g: $f_{UTP}(\langle \texttt{mel}(LHLH), LHLLLLLH \rangle) = LHHHHHHH$

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- E.g: $f_{UTP}(\langle \texttt{mel}(LHLH), LHLLLLLH \rangle) = LHHHHHHH$
- Note also that any string function can be converted into a multi-tier one

IML Functions and Automata

IML are modeled with asynchronous 1-way Deterministic Multi-tape Finite State Transducers (DMFSTs)

[Rawski and Dolatian, 2020, Furia, 2012, Elgot and Mezei, 1965, Rabin and Scott, 1959]

IML Functions and Automata

IML are modeled with asynchronous 1-way Deterministic Multi-tape Finite State Transducers (DMFSTs)

- FST with a set of states and transitions

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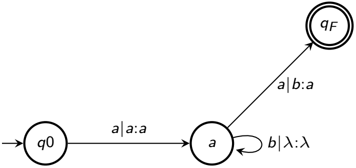

Figure: A Toy DM-FST.

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  - **Determinism:**
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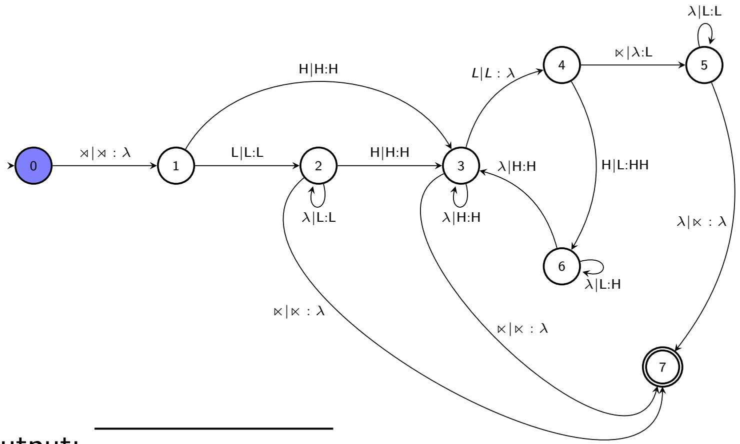
    - both input symbols cannot be empty strings (X or Y $\neq \lambda$)
  - **Locality:**
    - each state represents j-1 and k-1 factors, respectively on the melody and timing tapes
  - **Melody:**
    - both the melody and timing tapes share the same input alphabet ($X, Y \in \Sigma \cup \{\lambda\}$, where Σ={H,L})

Luganda Derivation: With Tone Plateauing

Input	mel(w)	⋊	L	H	L			H	⋉
	w	⋊	L	H	L	L	L	H	⋉



Output: ______________

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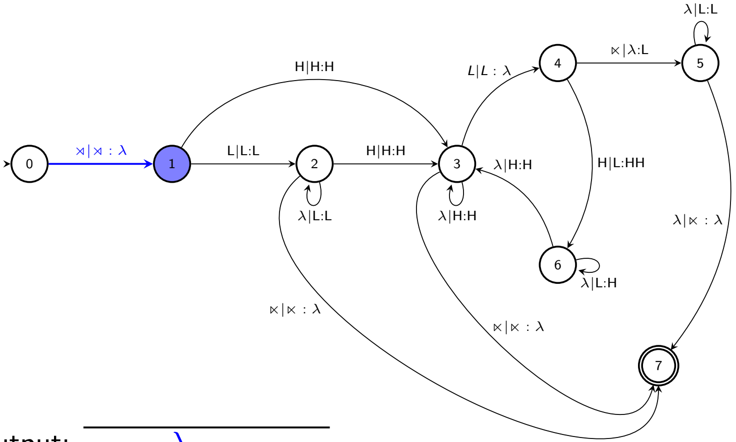
mel(w)	⋊	L	H	L			H	⋉
w	⋊	L	H	L	L	L	H	⋉

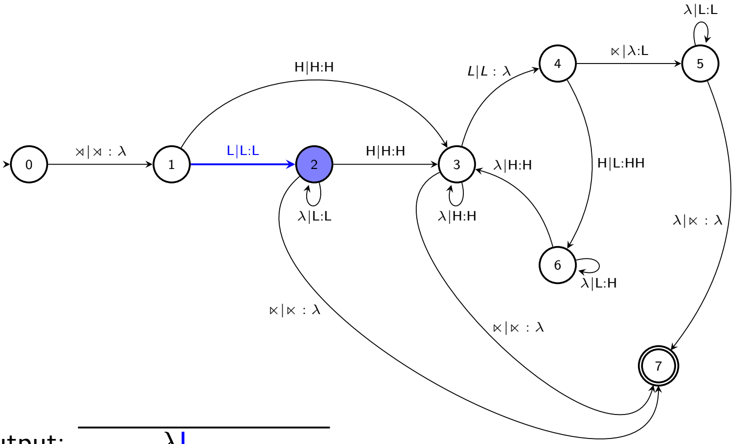


Output: λ

Luganda Derivation: With Tone Plateauing

Input

mel(w)	⋊	L	H	L			H	⋉
w	⋊	L	H	L	L	L	H	⋉



Output: λL

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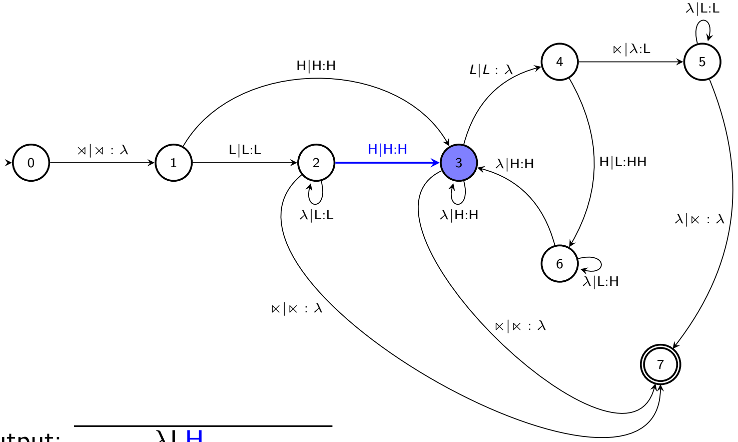
mel(w)	⋊	L	H	L			H	⋉
w	⋊	L	H	L	L	L	H	⋉

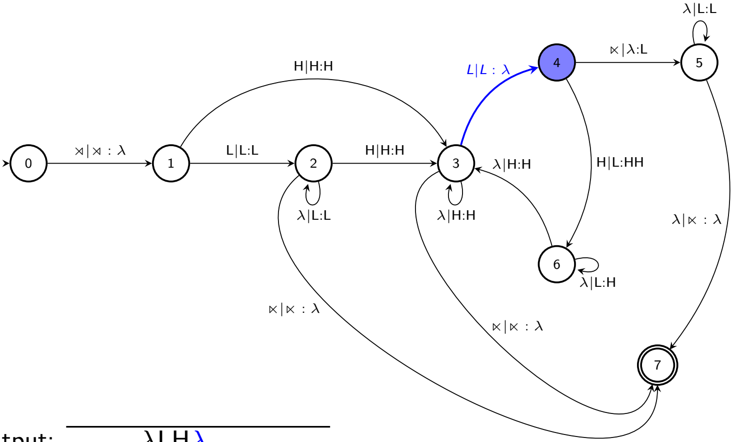


Output: λLH

Luganda Derivation: With Tone Plateauing

Input

mel(w)	⋊	L	H	L			H	⋉
w	⋊	L	H	L	L	L	H	⋉

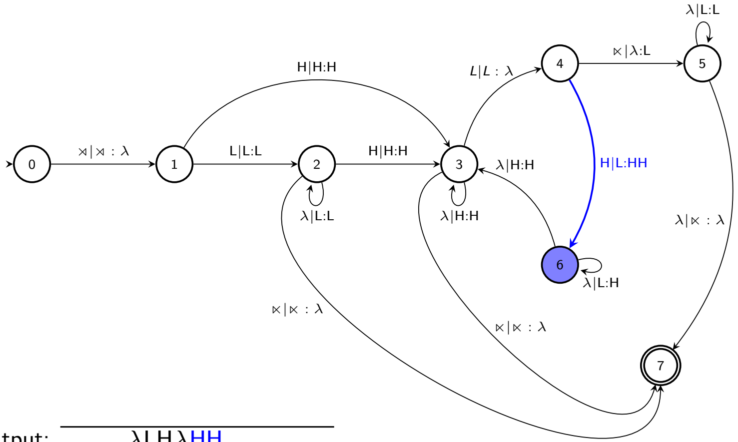


Output: λLHλ

Luganda Derivation: With Tone Plateauing

Input

mel(w)	⋊	L	H	L			H	⋉
w	⋊	L	H	L	L	L	H	⋉

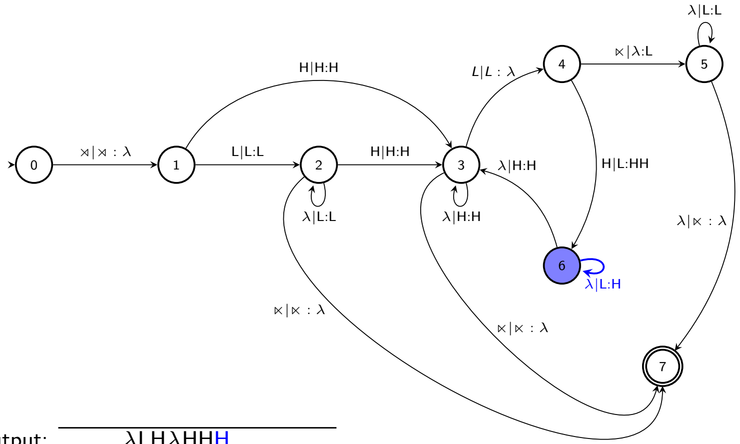


Output: λLHλHH

Luganda Derivation: With Tone Plateauing

Input

mel(w)	$\rtimes$	L	H	L			H	$\ltimes$
w	$\rtimes$	L	H	L	L	L	H	$\ltimes$

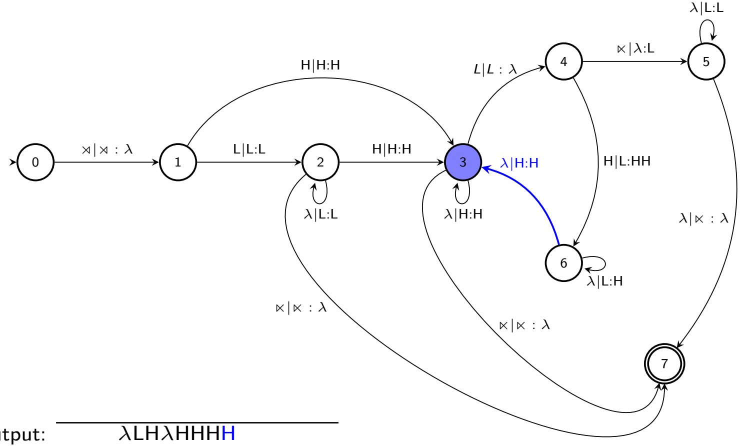


Output: λLHλHHH

Luganda Derivation: With Tone Plateauing

Input

mel(w)	⋊	L	H	L			H	⋉
w	⋊	L	H	L	L	L	H	⋉

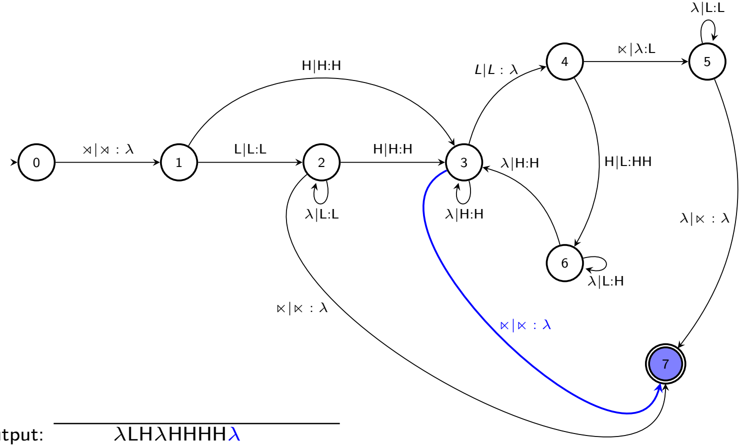


Output: λLHλHHHH

Luganda Derivation: With Tone Plateauing

Input

mel(w)	⋊	L	H	L			H	⋉
w	⋊	L	H	L	L	L	H	⋉

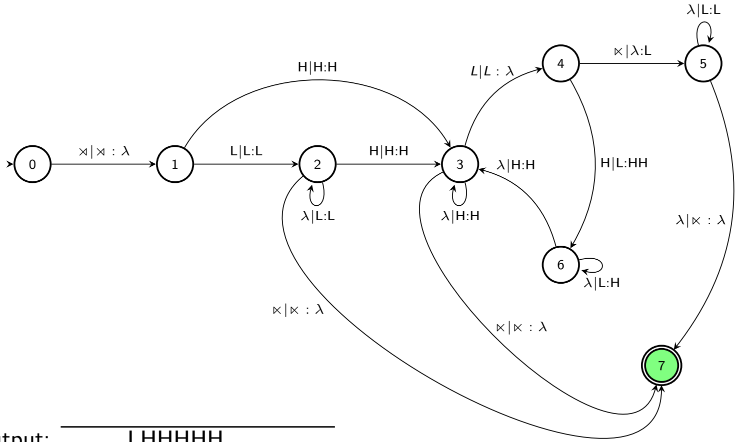


Output: λLHλHHHHλ

Luganda Derivation: With Tone Plateauing

Input

mel(w)	⋊	L	H	L			H	⋉
w	⋊	L	H	L	L	L	H	⋉

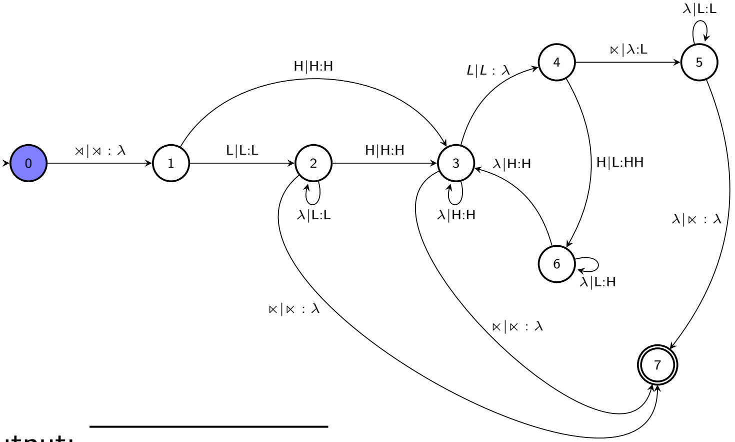


Output: LHHHHH

Luganda Derivation 2: Without Tone Plateauing

Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉

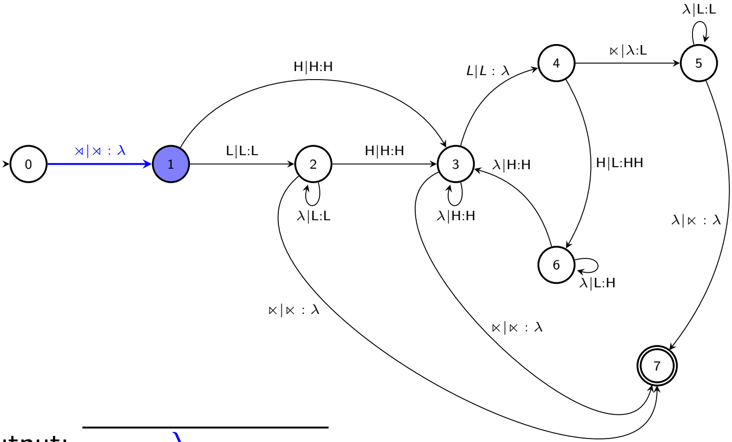


Output: ____________________

Luganda Derivation 2: Without Tone Plateauing

Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉

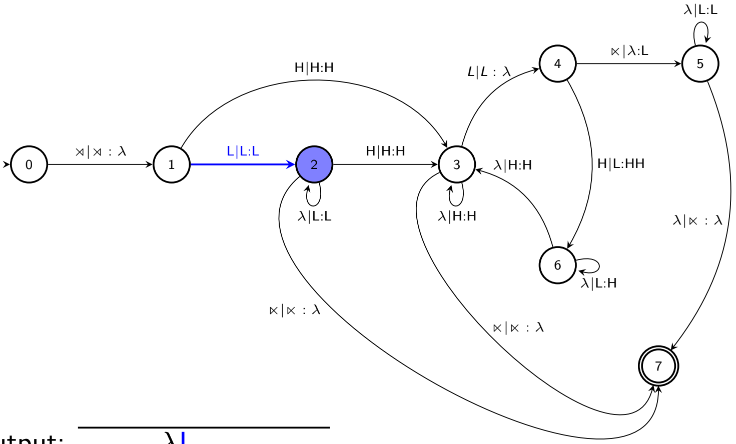


Output: λ

Luganda Derivation 2: Without Tone Plateauing

Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉

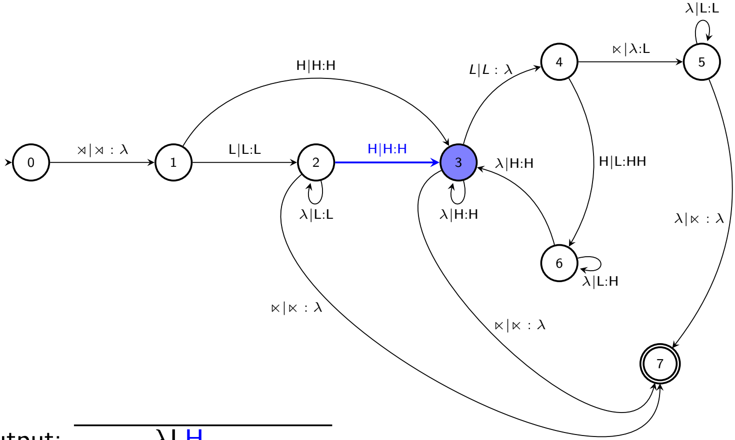


Output: λL

Luganda Derivation 2: Without Tone Plateauing

Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉

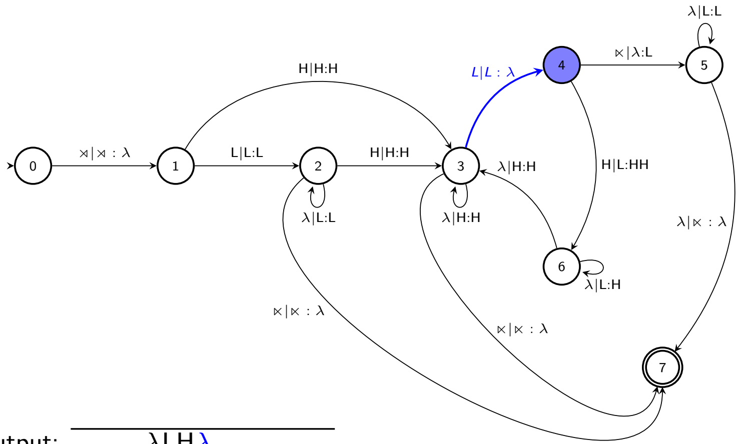


Output: λLH

Luganda Derivation 2: Without Tone Plateauing

Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉

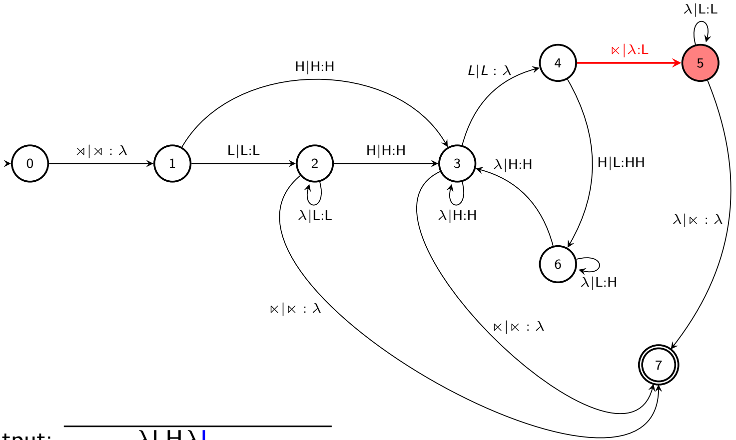


Output: λLHλ

Luganda Derivation 2: Without Tone Plateauing

Input

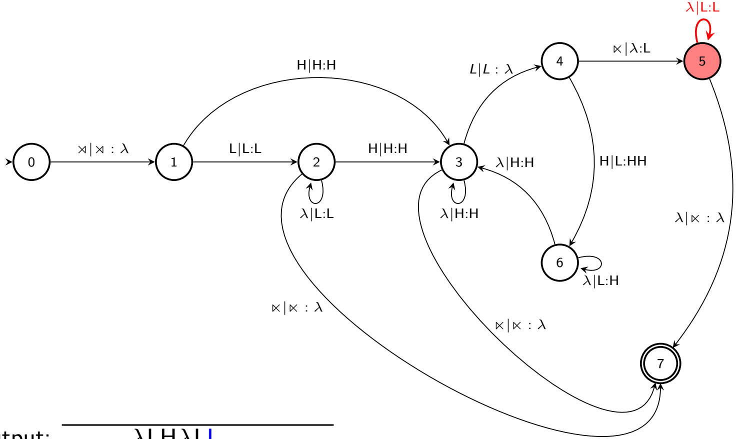
mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉



Output: λLHλL

Luganda Derivation 2: Without Tone Plateauing

Input								
mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉

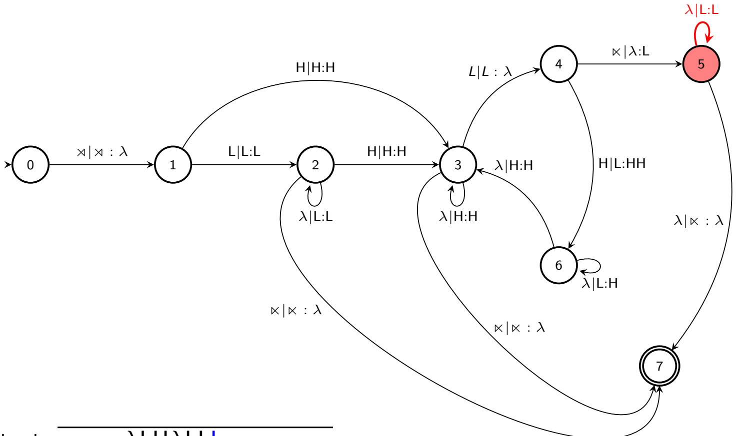


Output: λLHλLL

Luganda Derivation 2: Without Tone Plateauing

Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉



Output: λLHλLLL

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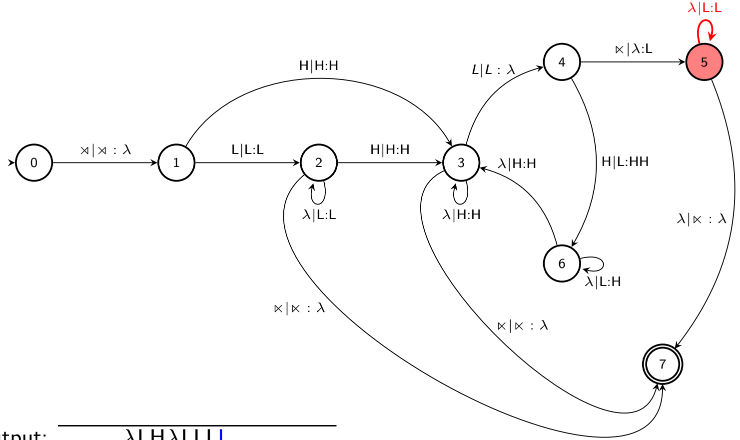
Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉



Output: λLHλLLLL

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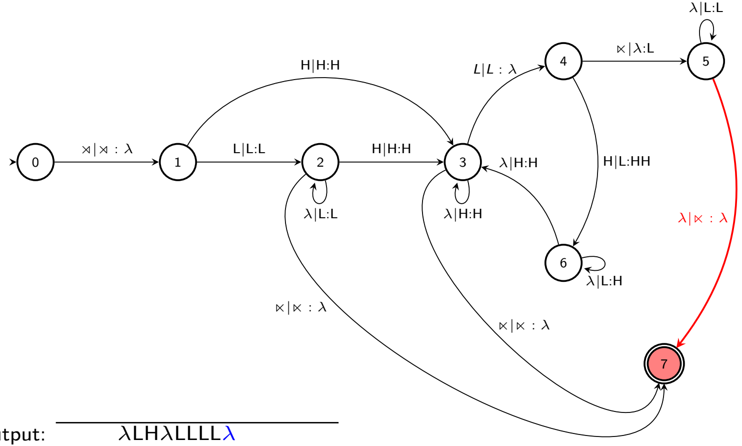
Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉

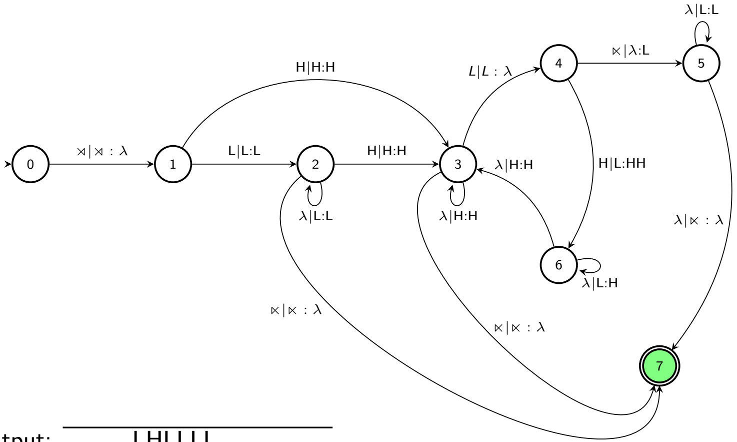


Output: λLHλLLLLλ

Luganda Derivation 2: Without Tone Plateauing

Input

mel(w)	⋊	L	H	L				⋉
w	⋊	L	H	L	L	L	L	⋉



Output: LHLLLL

Empirical Summary and Comparative Table

Tone Patterns and their Subregular Classes							
Patterns	Languages	ISL	OSL	A-ISL	Subseq	MISL	IML
Bounded shift	Rimi	✓	✗	✓	✓	✓	✓
Bounded Spread	Bemba	✓	✓	✓	✓	✓	✓
Bounded Meussen's Rule	Luganda	✓	✓	✗	✓	✓*	✓
Unbounded Shift	Zigula	✗	✗	✓	✓	✓	✓
Unbounded Spread	Ndebele	✗	✓	✗	✓	✓	✓
Unbounded Deletion	Arusa	✗	✗	✓	✓	✓	✓
Anticipatory downstep	Tiriki	✗	✓	✓	✓	-	✓
Anticipatory Upstep	Amo	✗	✗	✓(?)	✗	-	✓
UTP	Luganda	✗	✗	✗	✗	✓	✓
SG-like	C. Bemba	✗	✗	✗	✗	✓	✓
AMR	Shona	✗	✓	✗	✓	✗	✗
*Majority Rule[1]	-	-	-	-	-	-	✗
*Midpoint Pathology[2]	-	-	-	-	-	-	✗

[1] [Baković, 2000, Heinz and Lai, 2013]

[2] [Eisner, 1997]

Discussion

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  - which are also computed by deterministic asynchronous Multi-tape FSTs

[Rawski and Dolatian, 2020, Dolatian and Rawski, 2020]

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- Crucially, IML and MISL functions differ in two important regards:

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- Crucially, IML and MISL functions differ in two important regards:
  - Unlike with MISL, the input tapes of IML-computing DMFSTs are connected via a shared alphabet
  - Any IML function is also MISL, but not vice versa

[Rawski and Dolatian, 2020, Dolatian and Rawski, 2020]

Discussion

- The empirical results of the IML functions speak to 'phonological directionality' as well

[Rawski and Dolatian, 2020, Dolatian and Rawski, 2020, Zoll, 2003]

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- The empirical results of the IML functions speak to 'phonological directionality' as well
  - All of the patterns investigated fall in the intersection of L-IML and R-IML[3]

[Rawski and Dolatian, 2020, Dolatian and Rawski, 2020, Zoll, 2003]

[3] Though AU in Amo may be an exception.

Discussion

- The empirical results of the IML functions speak to 'phonological directionality' as well
  - All of the patterns investigated fall in the intersection of L-IML and R-IML[4]
  - Suggesting that directionality needs not be encoded in the grammar

[Rawski and Dolatian, 2020, Dolatian and Rawski, 2020, Zoll, 2003]

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Discussion

- The empirical results of the IML functions speak to 'phonological directionality' as well
  - All of the patterns investigated fall in the intersection of L-IML and R-IML[5]
  - Suggesting that directionality needs not be encoded in the grammar
- The Autosegmental Theory's Well-formedness conditions are preserved (for free):

[Zoll, 2003, Goldsmith, 1976, Williams, 1976]

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Discussion

- The empirical results of the IML functions speak to 'phonological directionality' as well
  - All of the patterns investigated fall in the intersection of L-IML and R-IML[6]
  - Suggesting that directionality needs not be encoded in the grammar
- The Autosegmental Theory's Well-formedness conditions are preserved (for free):
  - *No-gapping* constraint is satisfied by the melody function
  - *No-crossing* constraint is also satisfied through determinism and the melody

[Zoll, 2003, Goldsmith, 1976, Williams, 1976]

[6] Though AU in Amo may be an exception.

Future Research

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  - Their formal characterization is in progress

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- Expressivity of multi-tape FSTs
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- The Alternating Meussen's Rule is not IML, we conjecture it is Ouput Melody Local (OML)
  - OML is yet to be defined but can be based on [Chandlee, 2014] ' ISL/OSL
  - Suggests that tone functions are not all IML, but rather ML (IML&OML)
- IML functions, as currently formulated, work best for languages with underlying associations

Take-away Message

[Enriching the representation allows for a deterministic characterization of UC processes, a.o.]

Thanks!

Appendix 1: The two IML component functions

- **A Melody Function:** (Adapted from Jardine, 2020a)

$$\texttt{mel}(w) \stackrel{\text{def}}{=} \lambda \quad \text{if } w = \lambda,$$
$$\stackrel{\text{def}}{=} \texttt{mel}(v)\sigma \quad \text{if } w = v\sigma^n,\ v \neq u\sigma \text{ forsome} u \in \Sigma^*$$

- **The IO Function**: (e.g: UTP)

$$f_{utp}(\langle \texttt{mel}(w), w \rangle) \stackrel{\text{def}}{=} \text{L}^m H^{(2n+o)} \quad \text{if } w = \text{L}^m H^n L^o H^n,\ \texttt{mel}(w) = \text{(L)H(L)H};\ m \text{ \& o} \geq 0, \text{n} = 1$$
$$\stackrel{\text{def}}{=} w \quad \text{elsewhere.}$$

Appendix 2: A DMFST for Bounded Tone Shift in Rimi

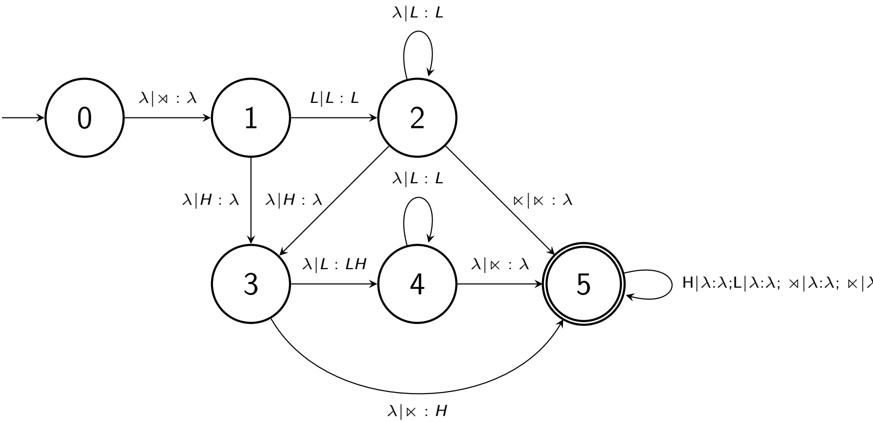

Figure: A 2-tape DM-FST for the one-step (bounded) tone shift in Rimi.

Appendix 3: Alternating Meussen's Rule in Shona

- Examples:

a.	/né-hóvé/	[né-hòvè]	'with-fish'
b.	/né-é-hóvé/	[né-è-hóvé]	'with-of-fish'
c.	/sé-né-é-hóvé/	[sé-nè-é hòvè]	'like-with-of-fish'

- AMR is not IML because the states of the DM-FST computing it do not represent $j-1$ and $k-1$ (input) factors.

[Odden, 1986, Chandlee and Jardine, 2019]

Appendix 4: Derivation Table for Luganda UTP

- For UTP, consider $w = \rtimes\mathrm{LHLLLH}\ltimes$ and melody $\mathtt{mel}(w) = \rtimes\mathrm{LHLH}\ltimes$

Step	Current state	Melody tape	Timing tape	Transition	Dest. state	Output
1.	*q0*	$\underline{\rtimes}\mathrm{LHLH}\ltimes$	$\underline{\rtimes}\mathrm{LHLLLH}\ltimes$	$\rtimes\|\rtimes{:}\lambda$	*q1*	
2.	*q1*	$\rtimes\underline{\mathrm{L}}\mathrm{HLH}\ltimes$	$\rtimes\underline{\mathrm{L}}\mathrm{HLLLH}\rtimes$	L|L:L	*q2*	L
3.	*q2*	$\rtimes\mathrm{L}\underline{\mathrm{H}}\mathrm{LH}\ltimes$	$\rtimes\mathrm{L}\underline{\mathrm{H}}\mathrm{LLLH}\rtimes$	H|H:H	*q3*	LH
4.	*q3*	$\rtimes\mathrm{LH}\underline{\mathrm{L}}\mathrm{H}\ltimes$	$\rtimes\mathrm{L}\underline{\mathrm{H}}\mathrm{LLLH}\rtimes$	$\mathrm{H}\|\lambda{:}\lambda$	*q4*	LH
5.	*q4*	$\rtimes\mathrm{LHL}\underline{\mathrm{H}}\ltimes$	$\rtimes\mathrm{LH}\underline{\mathrm{L}}\mathrm{LLH}\rtimes$	H|L:H	*q6*	LHH
6.	*q6*	$\rtimes\mathrm{LHL}\underline{\mathrm{H}}\ltimes$	$\rtimes\mathrm{LHL}\underline{\mathrm{L}}\mathrm{LH}\rtimes$	$\lambda\|\mathrm{L{:}H}$	*q6*	LHHH
7.	*q6*	$\rtimes\mathrm{LHL}\underline{\mathrm{H}}\ltimes$	$\rtimes\mathrm{LHLL}\underline{\mathrm{L}}\mathrm{H}\rtimes$	$\lambda\|\mathrm{L{:}H}$	*q6*	LHHHH
8.	*q6*	$\rtimes\mathrm{LHL}\underline{\mathrm{H}}\ltimes$	$\rtimes\mathrm{LHLLL}\underline{\mathrm{H}}\rtimes$	$\lambda\|\mathrm{H{:}H}$	*q3*	LHHHHH
9.	*q3*	$\rtimes\mathrm{LHLH}\underline{\ltimes}$	$\rtimes\mathrm{LHLLLH}\underline{\rtimes}$	$\ltimes\|\ltimes{:}\lambda$	*q7*	LHHHHH
10.	*q7*	$\rtimes\mathrm{LHLH}\ltimes$	$\rtimes\mathrm{LHLLLH}\rtimes$			LHHHHH

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